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LAMINAR CONVECTIVE HEAT TRANSFER IN HELICAL COILED TUBES

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Abstract—An experimental and numerical study has been made on convective heat transfer in coiled tubes. The experiments have been carried out for tube diameter/coil diameter ratios from 1/100 to 1/10, Prandtl numbers from 10 to 500 and Reynolds numbers from 20 to 4000. The heat transfer has been studied for two boundary conditions: for a uniform peripherally averaged heat flux and for a constant wall temperature. Attention has been paid to the heat transfer in the thermal entry region as well as in the fully developed thermal region. The results obtained and the relations proposed could be explained from and are based on the flow behaviour.

NOMENCLATURE

- d, tube diameter [m];
- D, coil diameter [m];
- Dn, Dean number [$\equiv Re(d/D)^{1/2}$];
- f, Weisbach friction factor;
- Gr, Grashof number ($\equiv \beta g \Delta T d^3/\nu^2$);
- Gz, Graetz number ($\equiv \langle w \rangle d^2/\alpha z$);
- h, local heat-transfer coefficient [W/m^2 °C];
- $\langle h \rangle$, overall heat-transfer coefficient [W/m^2 °C];
- L, tube length [m];
- Nu, local Nusselt number ($\equiv hd/\lambda$);
- $\overline{Nu}$, peripherally averaged Nusselt number;
- $\langle Nu \rangle$, overall Nusselt number;
- Pr, Prandtl number ($\equiv \nu/\alpha$);
- r, radial coordinate [m];
- Re, Reynolds number ($\equiv \langle w \rangle d/\nu$);
- T, temperature [°C];
- T', T^+, dimensionless temperature;
- $\langle T \rangle$, mean fluid temperature [°C];
- $\langle \Delta T \rangle$, logarithmic averaged temperature difference [°C];
- u, radial velocity [m/s];
- v, tangential velocity [m/s];
- w, axial velocity [m/s];
- z, axial coordinate [m].

Greek symbols

- α, thermal diffusivity [m^2/s];
- ζ, dimensionless radial coordinate;
- η, dynamic viscosity [kg/m s];
- λ, thermal conductivity [W/m °C];
- ν, kinematic viscosity [m^2/s];
- ξ, ξ^+, dimensionless axial coordinate;
- ϕ, tangential coordinate;
- Φ''_w, heat flux [W/m^2].

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INTRODUCTION

SECONDARY flow in coiled tubes as a result of centrifugal forces is a well-known phenomenon. This flow increases the heat and mass transfer as compared with the values obtained for straight tubes. The secondary flow pattern consists of two vortices perpendicular to the axial flow direction heat transport will take place not only by diffusion in the radial direction, but also by means of convection. The contribution of this convective heat transport is more or less dominating, depending on the flow conditions and fluid properties. Besides, this secondary flow also accounts for a considerable decrease in axial dispersion as compared with the straight tube. Nearly all research known with respect to heat transfer in coiled tubes dates from after 1950. Micheeff [2], Fostowskii [3], Kubair and Kuloor [4], Schmidt [5] and Shchukin [6] gave empirical relations for the overall heat-transfer coefficient in case of a constant wall temperature. Seban and McLaughlin [7], Dravid [8] and Singh and Bell [9] gave empirical relations for the local peripherally averaged heat-transfer coefficient for the fully developed thermal region in case of a uniform peripherally averaged heat flux. Mori and Nakayama [10] have tried to come to an analytical solution by analysing the velocity distribution and the temperature profile as well. More recently several numerical studies were made by Akiyama and Cheng [11, 12, 13], Tarbell and Samuels [14], Kalb and Seader [15, 16], and Patankar, Pratap and Spalding [17].

The different authors show that the various numerical methods do give results which are in good agreement with known experimental results. However attention has mainly been paid to situations with low Prandtl numbers only.

The heat-transfer relations as given by the various authors [7–9, 11–13, 15, 16] show a diversity in form even in case of the same boundary condition. This leads to significant differences in calculated heat-transfer coefficients calculated with these relations, especially in case of high Prandtl and Reynolds numbers. This often renders it very difficult to make a good prediction for the heat transfer in practical cases.

In general, hardly any attention has been paid to physical modelling from which experimental results could be explained or predictions made on the heat transfer under different conditions. The objective of this study was to arrive at a better understanding of the heat transfer in helical coiled tubes in case of laminar flow, in relation to the flow behaviour. The aim is to give heat-transfer relations based on both theoretical analysis and experiments over a wide range of Prandtl and Reynolds numbers and diameter ratios. Attention has also been paid to the effect of the boundary condition at the tube wall. Heat transfer experiments have been carried out with the same fluids and the same coiled pipes for two boundary conditions: a constant peripherally averaged heat flux and a uniform wall temperature. Also attention has been paid to both the heat transfer in the thermal entry region and the heat transfer in the fully developed thermal region.

FLOW CONSIDERATIONS

In order to gain a better insight into the relation between the heat transfer and the hydrodynamics an extensive study has been made of the literature on laminar flow in coiled tubes [17]. The main conclusions of this study, which are relevant for the explanation of the heat-transfer results are:

1. The hydrodynamics can be described with sufficient approximation by means of one characteristic dimensionless group, the Dean number: $Dn \equiv Re(d/D)^{1/2}$;

2. Three regions can be distinguished:

(a) the region of small Dean numbers, $Dn < 17$. In this region inertia forces due to the secondary flow can be neglected. The dimensionless secondary velocities ud/ν and vd/ν are proportional to Dn^2, the velocity distribution is satisfactorily described by the equations according to Dean [1];

(b) the region of intermediate Dean numbers, $17 < Dn < 100$. In this region the inertia forces due to the secondary flow balance the viscous forces more or less. The dimensionless secondary velocities are to a good approximation proportional to Dn;

(c) the region of high Dean numbers, $Dn > 100$. This region is characterised by a boundary layer flow, where only in the boundary layer near the tube wall the viscous forces are still significant. In the core region outside the boundary layer the dimensionless secondary velocities are approximately proportional to $Dn^{1/2}$. The velocity distribution is satisfactorily described by the equations according to Mori and Nakayama [10].

THEORETICAL CALCULATIONS

For the region of small Dean numbers the energy equation has been solved numerically, using the velocity distribution according to Dean [1]. The calculations have been carried out for two conditions:

(a) the fully developed thermal condition, in case of a uniform peripherally averaged heat flux; here asymptotic heat-transfer coefficients have been calculated;

(b) the boundary condition of a constant wall temperature; here the variations in local heat-transfer coefficients along the tube have been calculated.

For both situations a brief description will be given of the numerical procedure as carried out here [18].

(a) Stationary heat transport in the fully developed thermal region under the condition of a uniform peripherally averaged well heat flux

The stationary process of heat transport in laminar flow in a coiled tube can be described by the energy equation:

$$\alpha\nabla^2 T - w\frac{\partial T}{\partial z} - u\frac{\partial T}{\partial r} - v\frac{\partial T}{r\partial\phi} = 0, \quad (1)$$

where z, r and ϕ and w, u and v are the axial, radial and tangential coordinates and velocities, respectively (Fig. 1), α being thermal diffusivity.

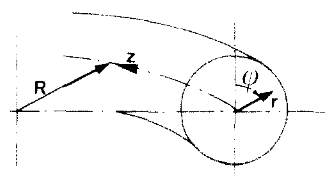

FIG. 1. Coordinates.

Having the boundary condition of a uniform peripherally averaged heat flux:

$$\frac{1}{\pi}\int_0^{\pi} \Phi''_w \, d\phi = \text{constant} \quad (2)$$

and a fully developed temperature profile, all temperatures will increase linearly with the axial distance z. Neglecting the diffusion in the axial direction and assuming that $\partial T/\partial z$ is independent of z, r and ϕ, equation (1) becomes two-dimensional in r and ϕ. Using the velocity distribution according to Dean [1], the dimensionless velocities can be given as:

$$u' = \frac{u}{2\langle w\rangle} = Re\frac{d}{D}F(\zeta,\phi) \quad (3)$$

$$v' = \frac{v}{2\langle w\rangle} = Re\frac{d}{D}G(\zeta,\phi) \quad (4)$$

$$w' = \frac{w}{2\langle w\rangle} \simeq (1-\zeta^2), \quad (5)$$

where $\zeta = 2r/d$ and F and G are functions only of the coordinates ζ and ϕ.

After substitution of the dimensionless velocities u', v' and w' and coordinates ζ and $\xi = 2z/d$, equation (1) can be written as:

$$\nabla^2_{\zeta\phi}T - Dn^2 Pr\left[F(\zeta,\phi)\frac{\partial T}{\partial\zeta} + G(\zeta,\phi)\frac{\partial T}{\zeta\partial\phi}\right] = (1-\zeta^2)RePr\frac{\partial T}{\partial\xi}. \quad (6)$$

Table 1.

Helix	Tube length (m)	Tube diameter (m)	Coil diameter (m)	d/D
No. 1	4.5	5.10^{-3}	5.10^{-1}	1.10^{-2}
2	5.8	1.10^{-2}	$6.2.10^{-1}$	$1.6.10^{-2}$
3	5.9	1.10^{-2}	$4.2.10^{-1}$	$2.4.10^{-2}$
4	5.5	1.10^{-2}	$1.2.10^{-1}$	$8.3.10^{-2}$

Table 2.

Helix	d/D	Pr	Dn	Re
1	1.10^{-2}	$30–1.10^2$	$5–2.10^2$	$50–2.10^3$
2	$1.6.10^{-2}$	$3.10^2–4.4.10^2$	2.5–50	$20–4.10^2$
3	$2.4.10^{-2}$	$27–4.4.10^2$	$3–6.10^2$	$20–4.10^3$
4	$8.3.10^{-2}$	40–80	$30–8.5.10^2$	$1.10^2–3.10^3$

Substituting $T' = T/RePr(\partial T/\partial \xi)$, e.g. (6) becomes:

$$\nabla^2_{\zeta,\phi}T' - Dn^2Pr\left[F(\zeta,\phi)\frac{\partial T'}{\partial \zeta} + G(\zeta,\phi)\frac{\partial T'}{\zeta\partial\phi}\right] = (1-\zeta^2). \quad (7)$$

As can be seen from equation (7), the dimensionless temperature distribution will be a function of the parameter Dn^2Pr only. Since equations (3), (4) and (5) are restricted to low Dean numbers ($Dn < 17$), it follows that Dn^2Pr is the characteristic group only in this region of Dean numbers.

The peripherally averaged Nusselt number in case of peripherally uniform wall temperature, will be found from the equation:

$$\overline{Nu} = 2RePr\frac{\partial T}{\partial \xi}\bigg/(T_w - \langle T\rangle) = 2/(T'_w - \langle T'\rangle). \quad (8)$$

The solution to equation (7) was quite straightforward, using a radial symmetrical grid and a five-point central difference scheme. The resulting set of finite difference equations was solved by the method of Gauss and Seidel.

(b) *Boundary condition of a constant wall temperature*

In this case equation (1) has to be solved step by step in the axial (z)-direction.

Substituting the dimensionless temperature:

$$T^+ = \frac{T - T_0}{T_w - T_0}, \quad (9)$$

where T_0 is the fluid temperature at the tube entry, and the dimensionless axial coordinate:

$$\xi^+ = \xi/RePr, \quad (10)$$

equation (1) can be written as:

$$(1-\zeta^2)\frac{\partial T^+}{\partial \xi^+} = \nabla^2_{\zeta,\phi}T^+ - Dn^2Pr\left[F(\zeta,\phi)\frac{\partial T^+}{\partial \zeta} + G(\zeta,\phi)\frac{\partial T^+}{\zeta\partial\phi}\right]. \quad (11)$$

Equation (4) was solved for every step $\Delta\xi^+$, using the Crank–Nicholson scheme for finite differences in the axial direction. The Nusselt number could be formed by the relation:

$$Nu = -\frac{2\partial T^+}{\partial \zeta}\bigg|_{\zeta-1}(T_w^+ - \langle T^+\rangle)^{-1}. \quad (12)$$

The grid used for the calculations consisted of (91) grid points, covering half a tube cross-section; 10 steps in the radial and 8 steps in the tangential direction. Owing to the occurrence of numerical instability, the numerical calculations could be made to Dn^2Pr values of up to about 6000.

It is useful to mention here that since the equations of mass transport and heat transport are the same, a more or less similar analysis can be made to calculate the axial dispersion in helical coiled tubes. This has been done in a previous publication [19].

EXPERIMENTAL SET UP

The heat-transfer experiments were carried out for two boundary conditions. First, the boundary condition of a uniform peripherally averaged heat flux, which was established by electric heating of the tubes. Secondly, the condition of an almost uniform wall temperature, which was established by heating with condensing steam. Both experimental set ups will be described.

(a) *Local heat-transfer measurement with the boundary condition of a uniform averaged heat flux*

The experiments were carried out with electrically heated coiled stainless steel tubes. The heating was obtained by using the tubewall as an electrical resistance. The averaged heat flux through the tube wall was in the range of 10^3–4.10^3 W/m^2. The temperature increase of the fluid was within the range of 5°C. Owing to these small temperature differences, the viscosity differences to 10°C were within the range of 40%. The experiments were carried out with four different coiled tubes (Table 1). The liquids used were water glycerol mixtures. The range of Re, Pr and Dn numbers in which the experiments were carried out are given in Table 2.

Table 3. Dimensions of tested coils

Helix	Tube length (m)	Tube diameter (m)	Coil diameter (m)	d/D
No. 1	5.7	1.10^{-2}	$1.5.10^{-1}$	$6.5.10^{-2}$
2	5.8	1.10^{-2}	$4.2.10^{-1}$	$2.4.10^{-2}$
3	5.5	5.10^{-3}	5.10^{-1}	1.10^{-2}

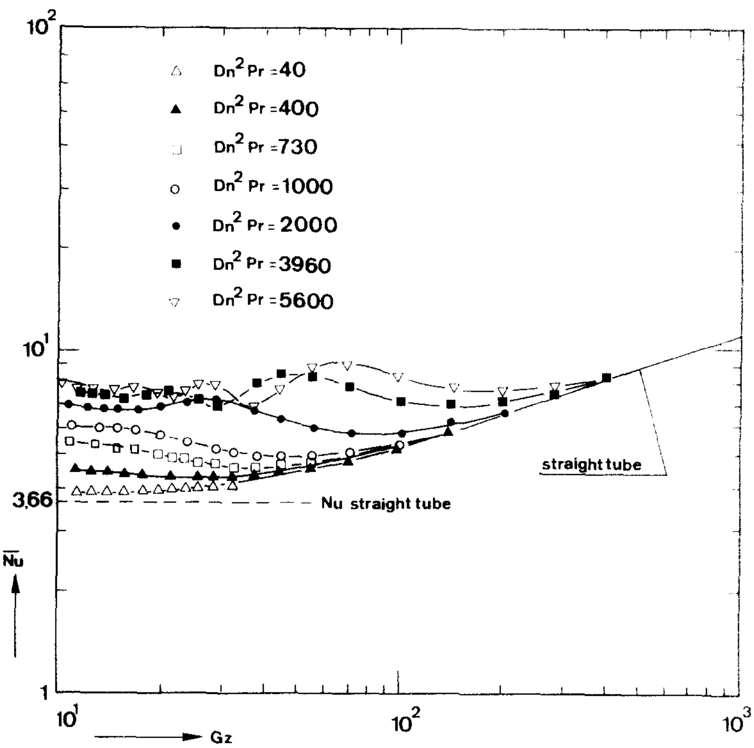

FIG. 2. Calculated values of Nu as a function of Gz with boundary condition T_w = constant.

Since the heat input per unit length of the tube is constant, the mean fluid temperature will rise linearly and is known in any axial place by measuring the inlet, and outlet temperatures of the fluid. The local heat-transfer coefficients were determined by means of equation (8) by measuring local wall temperatures.

The wall temperatures were measured with thermocouples at the outside of the tube wall, which was totally insulated by glass wool. The wall temperature was measured in various places in the axial as well as in the circumferential direction. The estimated accuracy of the local heat-transfer coefficients was 10–15%.

(b) *Overall heat-transfer measurement with the boundary condition of uniform wall temperature*

The constant wall temperature of 100°C established with condensing steam of 1 bar. The coiled tubes were placed in a closed stainless steel vessel, of which the steam pressure was regulated within 0.1 bar, and the steam temperature within 3°C. To determine a mean wall temperature the temperature was measured with four steam-insulated thermocouples in several places on the tube wall. The liquids used were two silicon oils with different viscosities, which were little temperature-dependent and a Shell-oil Vitrea 31, which had a more temperature-dependent viscosity. The experiments were carried out with three coiled tubes (Table 3).

The overall heat-transfer coefficients were calculated from the thermal balance:

$$\langle Nu\rangle_{\log} = \tfrac{1}{4}\langle RePr\rangle \frac{d}{L}\frac{\langle T\rangle_1 - \langle T\rangle_0}{\langle \Delta T\rangle_{\log}}. \qquad (13)$$

Where $\langle T\rangle_1$, is the averaged outlet temperature, $\langle T\rangle_0$ the averaged inlet temperature and $\langle \Delta T\rangle_{\log}$ the logarithmic averaged temperature difference between tube wall and fluid:

$$\langle \Delta T\rangle_{\log} = \frac{T_{w_0} - \langle T\rangle_0 - (T_{w_1} - \langle T\rangle_1)}{\left\{\dfrac{T_{w_0} - \langle T\rangle_0}{T_{w_1} - \langle T\rangle_1}\right\}}. \qquad (14)$$

The accuracy of the overall heat-transfer coefficients was estimated to be 10–20%.

RESULTS

(a) *Numerical results for small Dean numbers* (Dn < 17)

The results are given in Figs. 2 and 3. Figure 2 shows the results of the calculations for the boundary con-

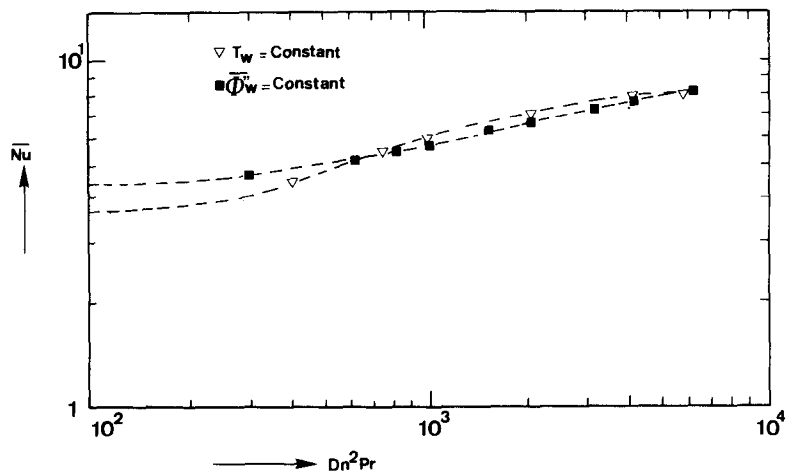

FIG. 3. Calculated values of $\overline{Nu}$ as a function of Dn^2Pr.

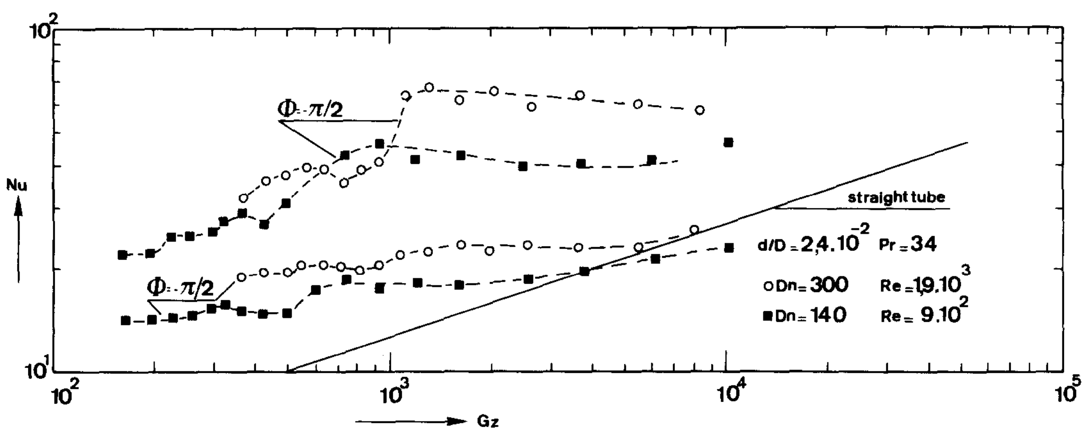

FIG. 4. Measured local Nu-numbers for $\phi = \pi/2$ and $\phi = -\pi/2$ as a function of Gz.

dition of a constant wall temperature, the peripherally averaged Nusselt number as a function of the Graetz number. The oscillating character of the curves is due to the effect of the secondary flow, which increases with increasing value of Dn^2Pr.

The "wavelength" of these oscillations is directly related to the ratio between the secondary velocities and axial velocity; this will be discussed in more detail under (b).

Figure 3 shows the asymptotic values of the peripherally averaged Nusselt number for the fully developed thermal region, both for the boundary condition of Φ''_w = constant and T_w = constant. These results fairly well agree with the numerical results of Akiyama and Cheng [11, 12, 13]. However, as mentioned already, only in the region of small Dean numbers can the heat transfer be characterised exclusively by the group Dn^2Pr. This aspect has not been mentioned by Akiyama and Cheng. It can be seen from Fig. 3 that the effect of the boundary condition becomes almost negligible with increasing value of Dn^2Pr unlike heat transfer in straight tubes, the heat transfer in case $\bar{\Phi}''_w$ = constant appears to become even a little lower than in case T_w = constant. Owing to the occurrence of free convection it was not possible to check these results with experimental ones in this particular region of Dn^2Pr values. The comparison with experimental results in an adjacent region will be discussed below.

(b) *Experimental results with the boundary condition of a uniform peripherally averaged heat flux*

An example of the results obtained for the local heat-transfer coefficient along the circumference of the helical tube is given in Fig. 4. The Nusselt numbers at the outside of the helix ($\phi = \pi/2$) and at the inside ($\phi = -\pi/2$) are given here as a function of the Graetz number. The variations in heat transfer at the outside ($\phi = \pi/2$) clearly reflect the oscillating character due to the circulating secondary flow. After every new circulation fluid of a higher temperature flows to the outer tube wall, which leads to a sudden decrease in temperature gradient at the tube wall and therefore a decrease in heat-transfer coefficient.

Since the thermal boundary layer at the outside of the helix ($\phi = \pi/2$) is thin, the heat transfer is very sensitive to temperature changes of the fluid, which is contrary to the inside of the helix ($\phi = -\pi/2$), where the thermal boundary layer is much thicker.

The axial distance z_s, where the heat transfer drops suddenly for the first time at $\phi = \pi/2$, can be roughly related to the ratio of mean axial and secondary velocity:

$$\frac{z_s}{d} \simeq \frac{\langle w \rangle}{\langle u \rangle}. \quad (15)$$

Since the fluid temperature at the outer tube wall will change when the fluid has crossed the tube cross-section along the symmetry line. Equation (15) can be written as:

$$\frac{\langle u \rangle d}{\nu} \simeq \frac{\langle w \rangle d}{\nu} \cdot \frac{d}{z_s}. \quad (16)$$

As pointed out before, the dimensionless secondary velocity $\langle u \rangle d/\nu$ is to a first approximation a function of the Dean number. In Fig. 5 the experimentally de-

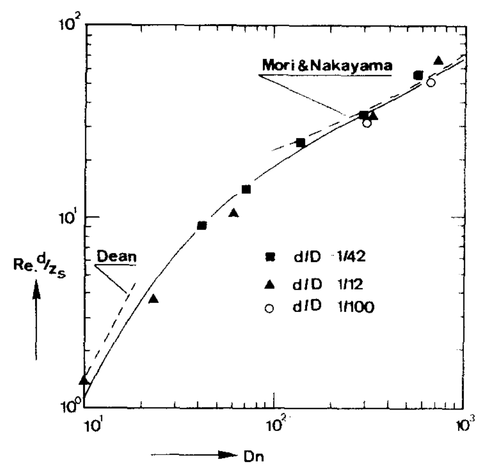

FIG. 5. Comparison between theoretical relations and experimentally determined secondary flow.

termined values of $Re d/z_s$ using the relation $\langle u \rangle d/\nu \simeq 10^{-2} Dn^2$ derived from the velocity distribution according to Dean [1] and $\langle u \rangle d/\nu \simeq 2Dn^{1/2}$ derived from the results of Mori and Nakayama [10]. Their good agreement shows that these analytically calculated velocity distributions give a good approximation for small and large Dean numbers respectively Fig. 6 gives some of the results for the peripherally averaged Nusselt numbers, calculated from the local values, as a function of the Graetz number. For the thermal entry region an empirical equation has been derived, which apart from the strong oscillating curve for very short tube length was within 20% in agreement with these experimental results:

$$\overline{Nu}_z = (0.32 + 3d/D) Re^{0.5} Pr^{0.33} \times (d/z)^{0.14 + 0.8d/D}. \quad (17)$$

For $20 < Dn < 8.3 \times 10^2$, $30 < Pr < 4.5 \times 10^2$ and $1 \times 10^{-2} < d/D < 8 \times 10^{-2}$. For the fully developed thermal region it appeared to be possible to correlate the peripherally averaged Nusselt number with the dimensionless axial velocity gradient:

$$\frac{\partial \bar{w}}{\partial r} \frac{d^2}{\nu} = \tfrac{1}{8} f Re^2. \quad (18)$$

Where f is the Weisbach function factor. This result is shown in Fig. 7. All results for $Dn > 20$ fitted within 10% the relation:

$$Nu = 0.43 (f Re^2)^{0.26} Pr^{1/6}. \quad (19)$$

For $20 < Dn < 8.3 \times 10^2$, $20 < Pr < 4.5 \times 10^2$ and $1 \times 10^{-2} < d/D < 8.3 \times 10^{-2}$.

Instead of equation (19) two asymptotic correlations were derived for the Nusselt number as a function of Re, Pr and d/D:

for $20 < Dn < 1 \times 10^2$

$$\overline{Nu} = 0.9 (Re^2 Pr)^{1/6} \quad (20)$$

for $1 \times 10^2 < Dn < 8.3 \times 10^2$

$$\overline{Nu} = 0.7 Re^{0.43} Pr^{1/6} (d/D)^{0.07}. \quad (21)$$

From equations (20) and (21) it can be seen that the effect of the ratio d/D on the Nusselt number was negligible for $20 < Dn < 1 \times 10^2$ and only very small for $Dn > 1 \times 10^2$. The result found experimentally (equation 19) that the peripherally averaged Nusselt number can be correlated with the dimensionless axial velocity gradient at the tube wall (proportional to f) can be explained from the analogy with the Lévêque equation for the heat transfer in case of a constant shear stress layer. Lévêque predicts for that case a heat-transfer coefficient proportional to $(\partial \bar{w}/\partial r)^{1/3}$.

Since fluid is streaming continuously from the tube centre to the tube wall and only for a certain period of time along the tube wall during each circulation, the heat transfer will remain dependent on the velocity gradient at the tube wall, even in the fully developed thermal region.

Though the exact proportionality of the Nusselt number to $(\partial \bar{w}/\partial r)^{1/3}$ was not found, the correlation between $(\partial \bar{w}/\partial r)$ and $\overline{Nu}$ was found to be consistent over the total range of Dean numbers for $Dn > 20$.

The results for $Dn < 20$ are shown in Fig. 8, from which one can see that they match the numerical results fairly well. An empirical relation

$$\overline{Nu} = 1.7 (Dn^2 Pr)^{1/6} \quad (22)$$

was derived for $Dn < 20$ and $(Dn^2 Pr)^{1/2} > 1 \times 10^2$.

Two main conclusions can be drawn from the results as described by equations (19)–(22). In the first place it appeared clearly from the experiments that the peripherally averaged asymptotic Nusselt number for the fully developed thermal region can be described as a function of $Dn^2 Pr$ in case of small Dean numbers only. For $Dn > 20$ the Reynolds and d/D dependency of the asymptotic Nusselt numbers appeared not to be described by the Dean number as suggested by Dravid [8], Akiyama and Cheng [11–13] and Kalb and Seader [15, 16].

Secondly the Prandtl dependency of the asymptotic Nusselt number appeared to be described by $Pr^{1/6}$ for

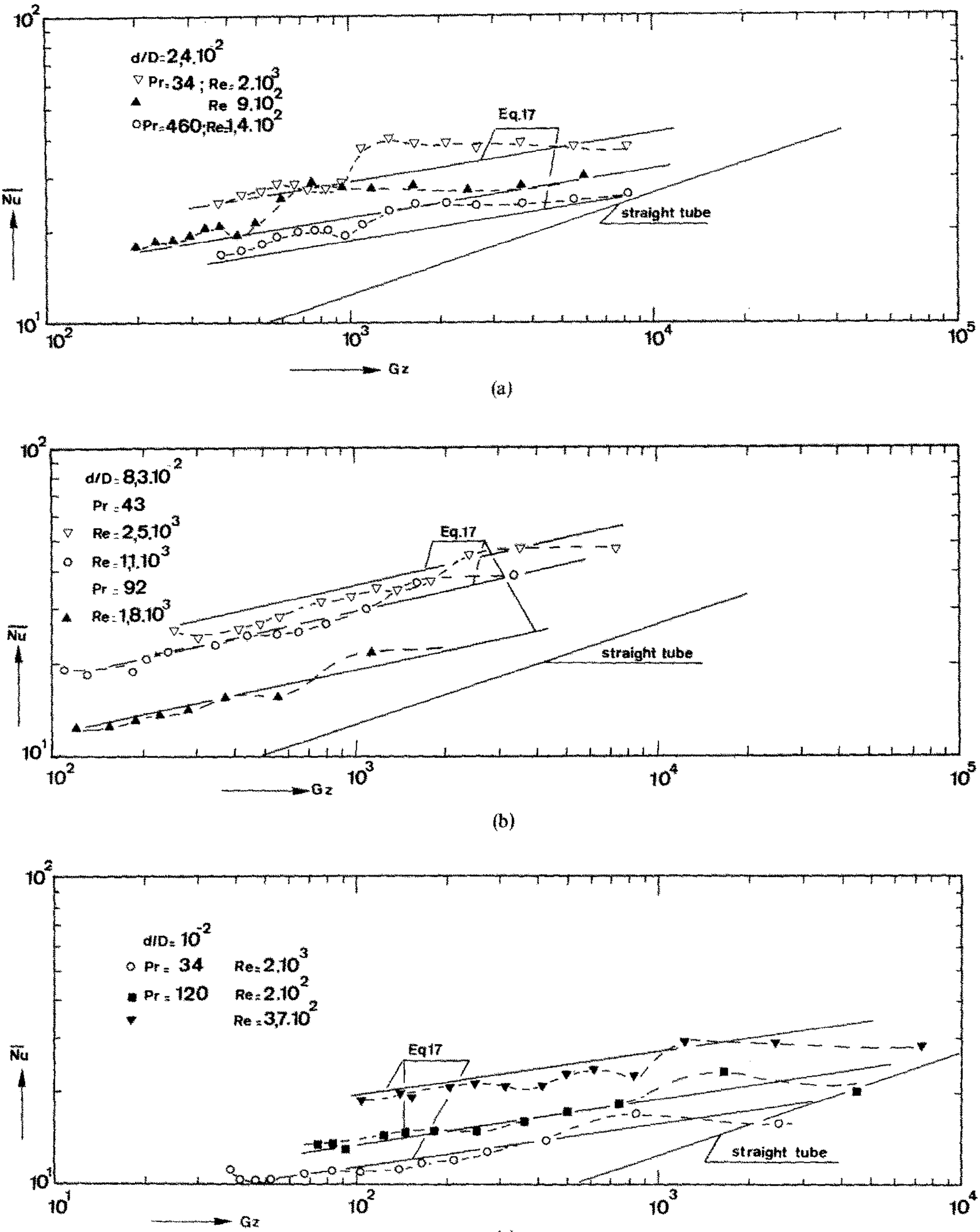

FIG. 6. Measured values of $\overline{Nu}$ as a function of Gz.

all experiments, which is in agreement with the experimental results of Dravid [8]. This was not found by Seban and McLanghlin [7] and Bell and Singh [9], who both give a Prandtl dependency of $Pr^{1/3}$, which was found from our experiments only to be the case in the thermal entry region.

Finally, for the length of the thermal entry region, which has been defined as the region where the heat transfer differs more than 15% from the asymptotic value, a rough relation has been found:

$$z/d \leqslant 20(d/D)^{-0.5}(Pr)^{0.2} \quad (23)$$

or

$$Gz \geqslant 5 \times 10^{-2} Dn Pr^{0.8}. \quad (24)$$

From equation (23) it can be concluded that the thermal entry length is mainly determined by a certain number of secondary flow circulations and little by the

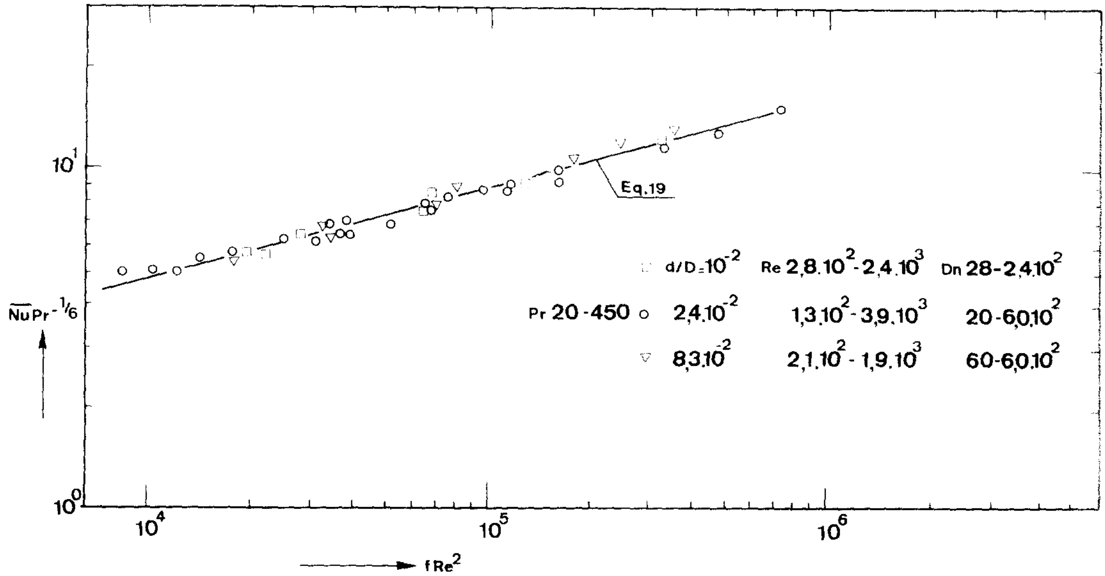

FIG. 7. Measured asymptotic $\overline{Nu}$ numbers as a function of fRe^2.

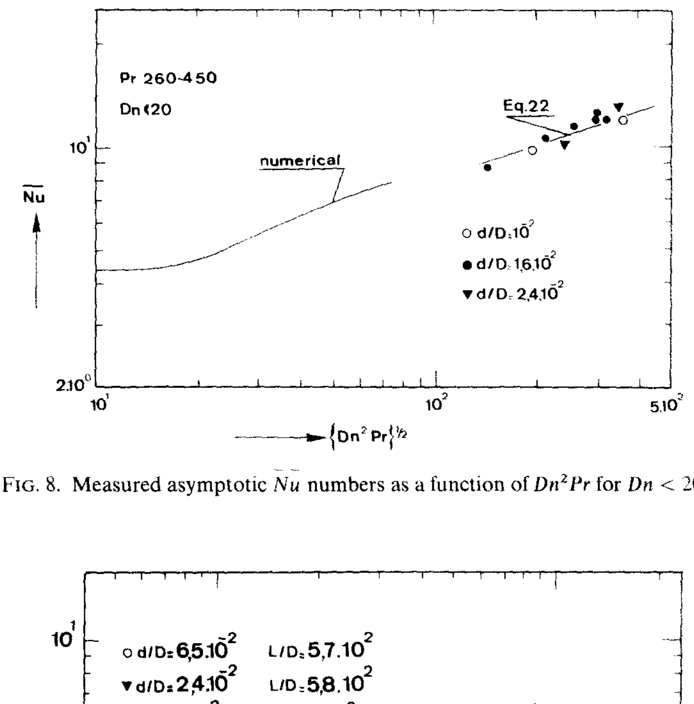

FIG. 8. Measured asymptotic $\overline{Nu}$ numbers as a function of Dn^2Pr for $Dn < 20$.

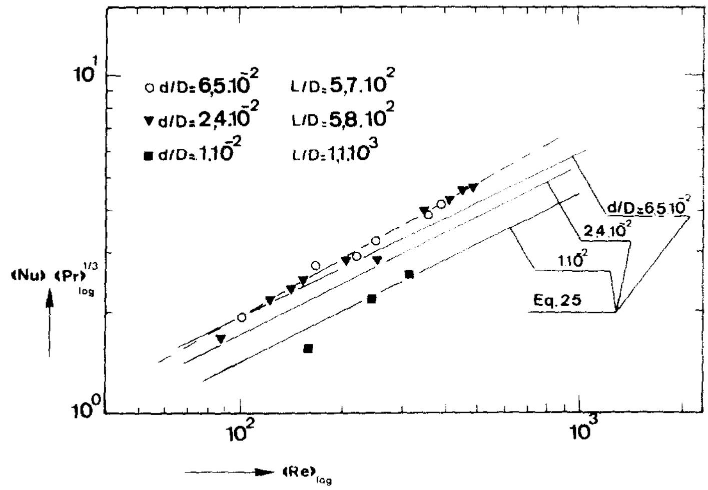

FIG. 9. Measured values of $\langle Nu \rangle$ as a function of $\langle Re \rangle_{\log}$ for different values of d/D.

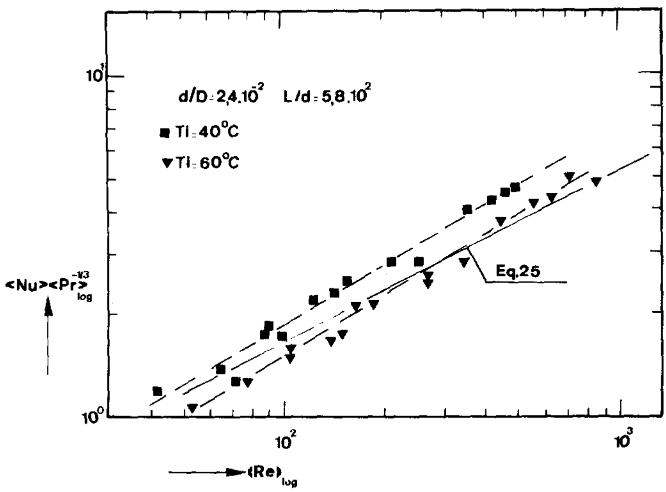

FIG. 10. Measured values of $\langle Nu \rangle_{\log}$ as a function of $\langle Re \rangle_{\log}$ for different inlet temperatures.

effect of thermal diffusivity, as it is in straight tubes. Depending on the values of Re, Prandtl and d/D the thermal entry length was 20–90% smaller than in straight tubes.

(c) *Experimental results with the boundary condition of a constant wall temperature*

The overall Nusselt numbers obtained have been compared with equation (17) integrated over the tube length:

$$\langle Nu \rangle = \frac{1}{L}\int_0^L \overline{Nu}\,\mathrm{d}z = \left(\frac{0.32+3d/D}{0.86-0.8d/D}\right) \times Re^{0.5}Pr^{0.33}(d/L)^{0.14+0.8d/D} \quad (25)$$

This relation however neglects non-isoviscous effects. The results for different oils are given in Figs. 9 and 10. It appeared that the effect of the non-isoviscous flow on the heat transfer could not be described by the Sieder–Tate connection $(\langle \eta \rangle / \eta_w)^{0.14}$.

The temperature depending appeared to be larger than predicted by this correction. As can be seen from Fig. 10, the results are clearly affected by the fluid inlet temperature. On the other hand, no significant differences were found between the silicon oils and the vitrea oil with a more temperature dependent viscosity. In case of not too large temperature differences ($T_0 = 60°C$) the overall Nusselt numbers obtained show reasonable agreement with equation (25). Therefore it may be concluded that there is little effect of the kind of boundary condition on the heat transfer. However, the heat-transfer coefficients will be strongly effected by the absolute temperature differences between tube wall and fluid.

SUMMARY AND CONCLUSIONS

From the results of this study it has appeared that only for low values of the Dean number ($Dn < 20$) the dimensionless group Dn^2Pr is characteristic for the heat transfer.

For $Dn > 20$ it has been shown that the heat-transfer coefficient in the fully developed thermal region is directly related to the mean axial shear rate at the tube wall. For $20 < Dn < 100$ it is shown that the effect of the value of the diameter ration d/D can be neglected for the fully developed thermal region, the Nusselt number can be described as a function of the group Re^2Pr only.

For all cases with $[Dn^2Pr]^{1/2} > 100$ the Nusselt number in the fully developed thermal region is proportional to $Pr^{1/6}$.

For the thermal entry region the Prandtl dependency of the Nusselt number appeared to be better described by $Pr^{1/3}$. It has been found that the length of the thermal entry region is mainly determined by a certain number of secondary flow circulation necessary to establish the temperature distribution.

A comparison of the overall heat-transfer coefficients in case of a constant wall temperature and a constant averaged heat flux shows the effect of the boundary condition to be small, provided the flow can be considered as isoviscous. Thereby it has been found that the non-isoviscous flow effects on the heat transfer is larger than predicted by the Sieder–Tate correction.

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CONVECTION THERMIQUE LAMINAIRE DANS DES TUBES EN SERPENTIN

Résumé—Il s'agit d'une étude expérimentale et théorique de la convection thermique dans les tubes cintrés hélicoïdalement. Les expériences sont relatives à des rapports du diamètre du tube au diamètre du serpentin variant de 1/100 à 1/10, à un nombre de Prandtl allant de 10 à 500 et à des nombres de Reynolds compris entre 20 et 4000. Le transfert de chaleur a été étudié pour deux conditions aux limites: avec un flux thermique circonférentiel moyen constant et avec une température pariétale constante. L'attention a été portée sur le transfert thermique dans la région d'entrée aussi bien que dans la région de l'établissement de régime thermique. Les résultats obtenus et les formules proposées sont expliquées en relation avec la configuration de l'écoulement.

LAMINAR KONVEKTIVER WÄRMEÜBERGANG IN SPIRALFÖRMIG GEWICKELTEN ROHREN

Zusammenfassung—Eine experimentelle und theoretische Studie zur Bestimmung des konvektiven Wärmeübergangs in Spiralrohren wurde durchgeführt. Die Versuche wurden für Rohrdurchmesser/Wirbeldurchmesser-Verhältnisse von 1/100 bis 1/10, Prandtl-Zahlen von 10 bis 500 und Reynolds-Zahlen von 20 bis 4000 durchgeführt. Der Wärmeübergang wurde für zwei Randbedingungen untersucht, einerseits für einen gleichförmigen, in Umfangsrichtung gemittelten Wärmestrom und andererseits für konstante Wandtemperatur. Besonders beachtet wurde der Wärmeübergang sowohl in der thermischen Einlaufzone als auch im Gebiet thermisch vollständig ausgebildeter Strömung. Die gewonnenen Ergebnisse und die vorgeschlagenen Beziehungen konnten aus dem Strömungsverhalten erklärt bzw. daraus abgeleitet werden.

ПЕРЕНОС ТЕПЛА ЛАМИНАРНОЙ КОНВЕКЦИЕЙ В СПИРАЛЬНЫХ ТРУБКАХ

Аннотация — Проведено экспериментальное и численное исследование конвективного переноса тепла в спиральных трубках. Эксперименты проводились при отношениях диаметра трубки к диаметру спирали равных, 1/100–1/10, числах Прандтля в диапазоне 10–500 и числах Рейнольдса в диапазоне 20–4000. Исследовался теплообмен для двух граничных условий: однородного по окружности теплового потока и постоянной температуры стенки. Особое внимание обращалось на перенос тепла в тепловом начальном участке, а также в области стабилизированного теплообмена. Полученные результаты и соотношения можно объяснить, исходя из характера течения.